Practica 2

Limites

Problema 1

En los siguientes casos calcule el valor del limite.

a) $$\lim_{(x,y)\to(0,0)} \frac{3\,x^2 - y^2 + 5}{x^2 + y^2 + 2}$$

b) $$\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}}$$

c) $$\lim_{(x,y)\to(2,-3)} \left(\frac{1}{x} + \frac{1}{y}\right)^2$$

d) $$\lim_{(x,y)\to(0,\frac{\pi}{4})} \sec(x)\operatorname{tg}(y)$$

a)

Por "evaluación"

$$\lim_{(x,y)\to(0,0)} \frac{3\,x^2 - y^2 + 5}{x^2 + y^2 + 2} = \frac{3\,(0)^2 - (0)^2 + 5}{(0)^2 + (0)^2 + 2} = \frac{5}{2}$$

Si usamos una familia de curvas: $y = k\,x$

$$\lim_{x\to 0} = \frac{3\,x^2 - (k\,x)^2 + 5}{x^2 + (k\,x)^2 + 2} = \frac{5}{2}$$

b) Por "evaluación"

$$\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}} = \frac{(0)}{\sqrt{(4)}} = 0$$

Si usamos una familia de curvas:

$$y = k\,x^2 + 4$$

$$\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}} = \lim_{x\to 0} \frac{x}{\sqrt{k\,x^2+4}} = 0$$

c) Por "evaluación"

$$\lim_{(x,y)\to(2,-3)} \left(\frac{1}{x}+\frac{1}{y}\right)^2 = \left(\frac{1}{(2)}+\frac{1}{(-3)}\right)^2 = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

Si usamos una familia de curvas: $y = k(x-2)-3$

$$\lim_{(x,y)\to(2,-3)} \left(\frac{1}{x}+\frac{1}{y}\right)^2 = \lim_{x\to 2} \left(\frac{1}{x}+\frac{1}{(k(x-2)-3)}\right)^2 = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

e) Por "evaluación"

$$\lim_{(x,y)\to(0,\frac{\pi}{4})} \sec(x)\text{tg}(y) = \sec(0)\text{tg}(\frac{\pi}{4}) = 1$$

Si usamos una familia de curvas: $y = k\,x + \frac{\pi}{4}$

$$\lim_{(x,y)\to(0,\frac{\pi}{4})} \sec(x)\text{tg}(y) = \lim_{x\to 0} \sec(x)\text{tg}(k\,x + \frac{\pi}{4}) = 1$$

Problema 2.

Calcular:

a) $$\lim_{(x,y)\to(1,1)} \frac{x^2-2xy+y^2}{x^2+y^2+2}$$

b) $$\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1}$$

c) $$\lim_{(x,y)\to(1,1)} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$$

d) $$\lim_{(x,y)\to(0,0)} \frac{x+y-4}{\sqrt{x+y}-2}$$

Ejercicio 1

a)

$$\lim_{(x,y)\to(1,1)} \frac{x^2 - 2xy + y^2}{x^2 + y^2 + 2} = \lim_{(x,y)\to(1,1)} \frac{(x-y)^2}{x-y} = \lim_{(x,y)\to(1,1)} (x-y) = 0$$

$x \neq y$!

b)

$$\lim_{(x,y)\to(1,1)} \frac{xy - y - 2x + 2}{x-1} = \lim_{(x,y)\to(1,1)} \frac{(x-1)(y-2)}{x-1} = \lim_{(x,y)\to(1,1)} (y-2) = -1$$

$x \neq 1$!

c)

$$\lim_{(x,y)\to(0,0)} \frac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}} = \lim_{(x,y)\to(0,0)} \frac{\left(\sqrt{x} - \sqrt{y}\right)\left(\sqrt{x} + \sqrt{y} + 2\right)}{\sqrt{x} - \sqrt{y}}$$

$x \neq y$ **!**

$$\lim_{(x,y)\to(0,0)} \frac{\left(\sqrt{x} - \sqrt{y}\right)\left(\sqrt{x} + \sqrt{y} + 2\right)}{\sqrt{x} - \sqrt{y}} = \lim_{(x,y)\to(0,0)} \left(\sqrt{x} + \sqrt{y} + 2\right)$$

$$\lim_{(x,y)\to(0,0)} \left(\sqrt{x} + \sqrt{y} + 2\right) = 2$$

Problema 3.

Calcular:

a) $$\lim_{P\to(1,2,3)} \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

Ejercicio 2-3

b) $$\lim_{P\to(1,-1,-1)} \frac{2xy+yz}{x^2+z^2}$$

Problema 4.

Calcular los siguientes limites:

a) $$\lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}}$$

b) $$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$$

Ejercicio 4

c) $$\lim_{(x,y)\to(0,0)} \frac{x\,y}{|x\,y|}$$

a)

Tomamos la curva $y = x$ y ademas $x > 0$

$$\lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}} = \lim_{x\to 0^+} -\frac{x}{\sqrt{x^2+(x)^2}}$$

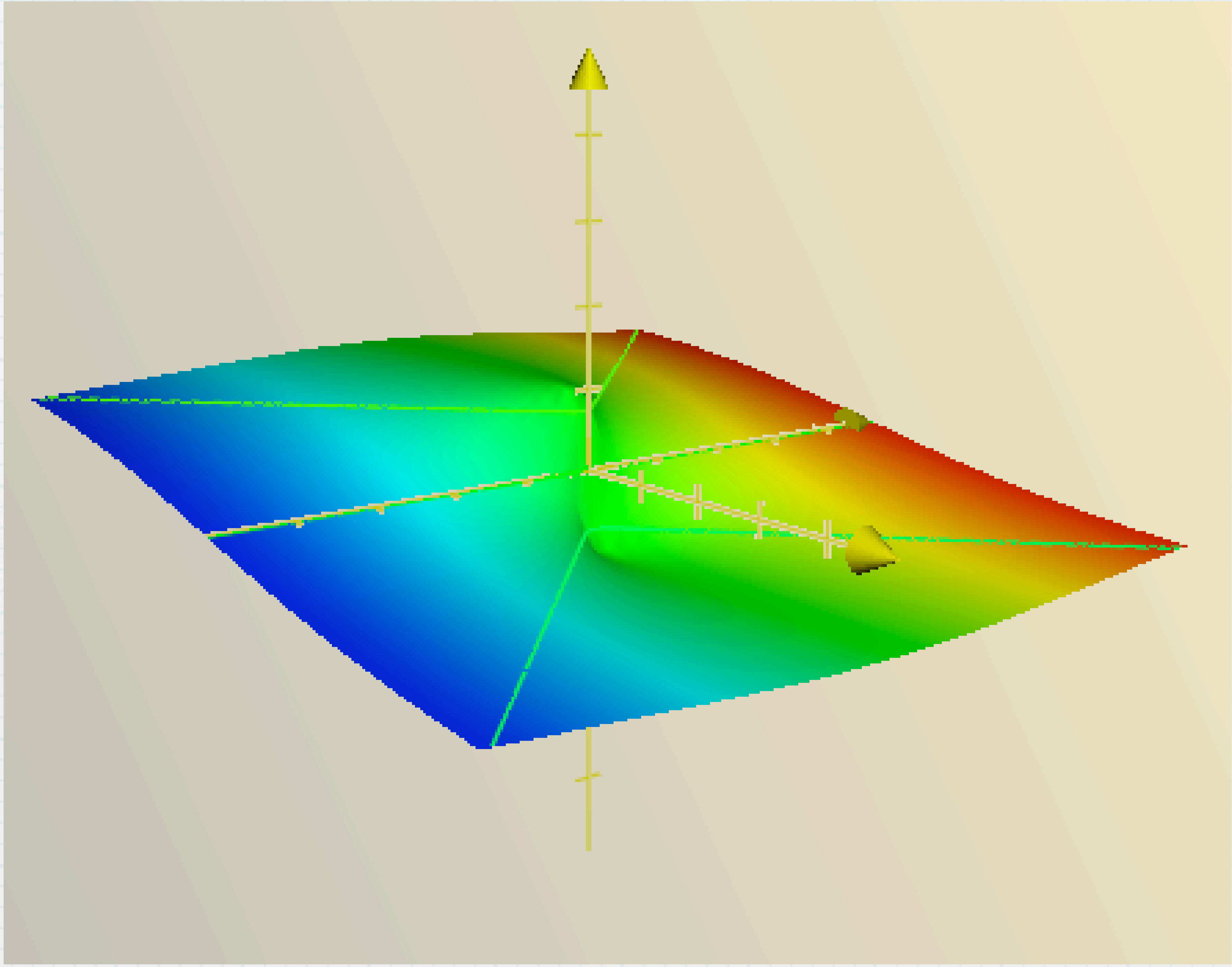
$$\lim_{x\to 0^+} -\frac{x}{\sqrt{x^2+(x)^2}} = \lim_{x\to 0^+} -\frac{x}{\sqrt{2}\,|x|} = \lim_{x\to 0^+} -\frac{x}{\sqrt{2}x} = \lim_{x\to 0^+} -\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

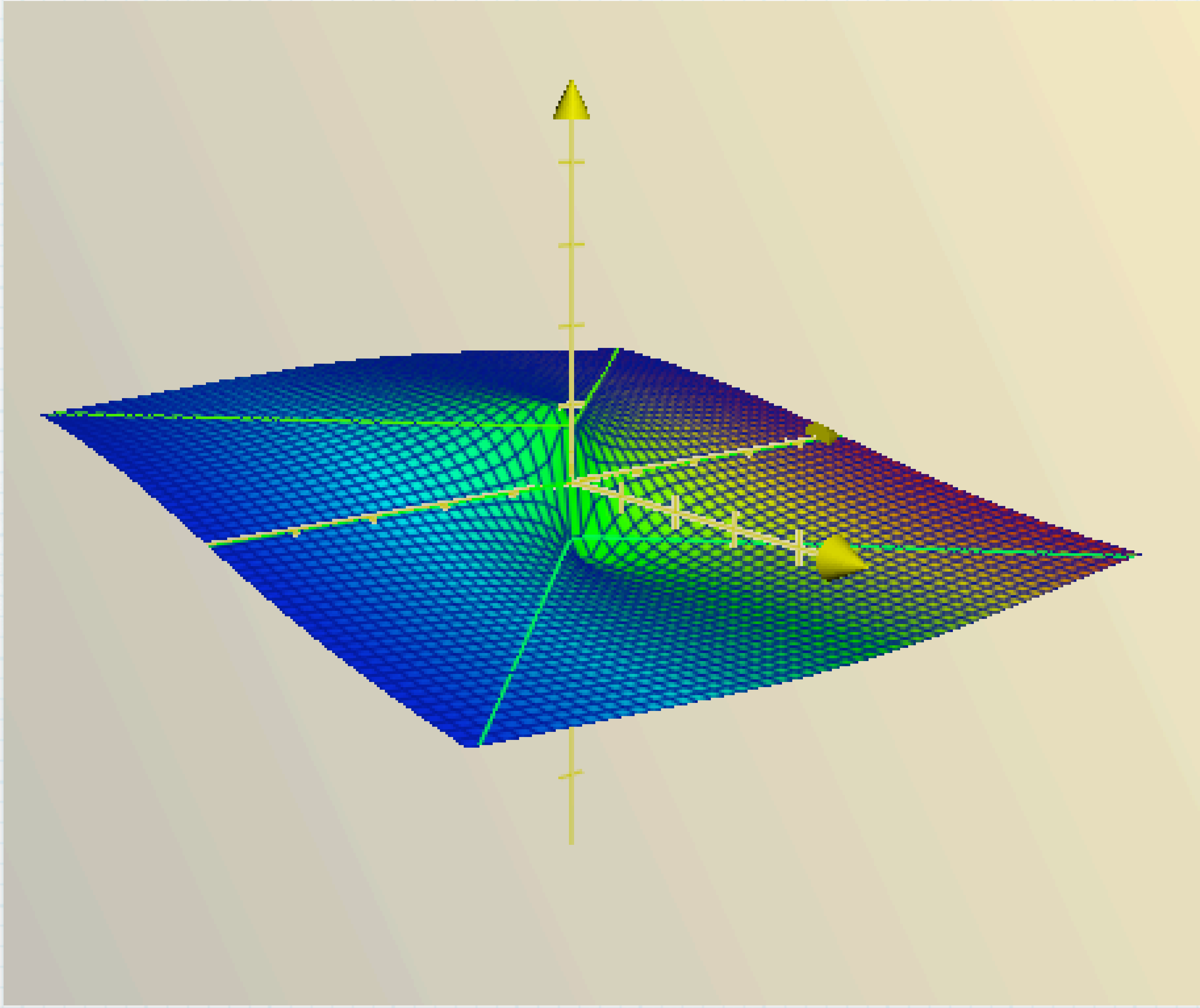
Tomamos la curva $y = x$ y ademas $x < 0$

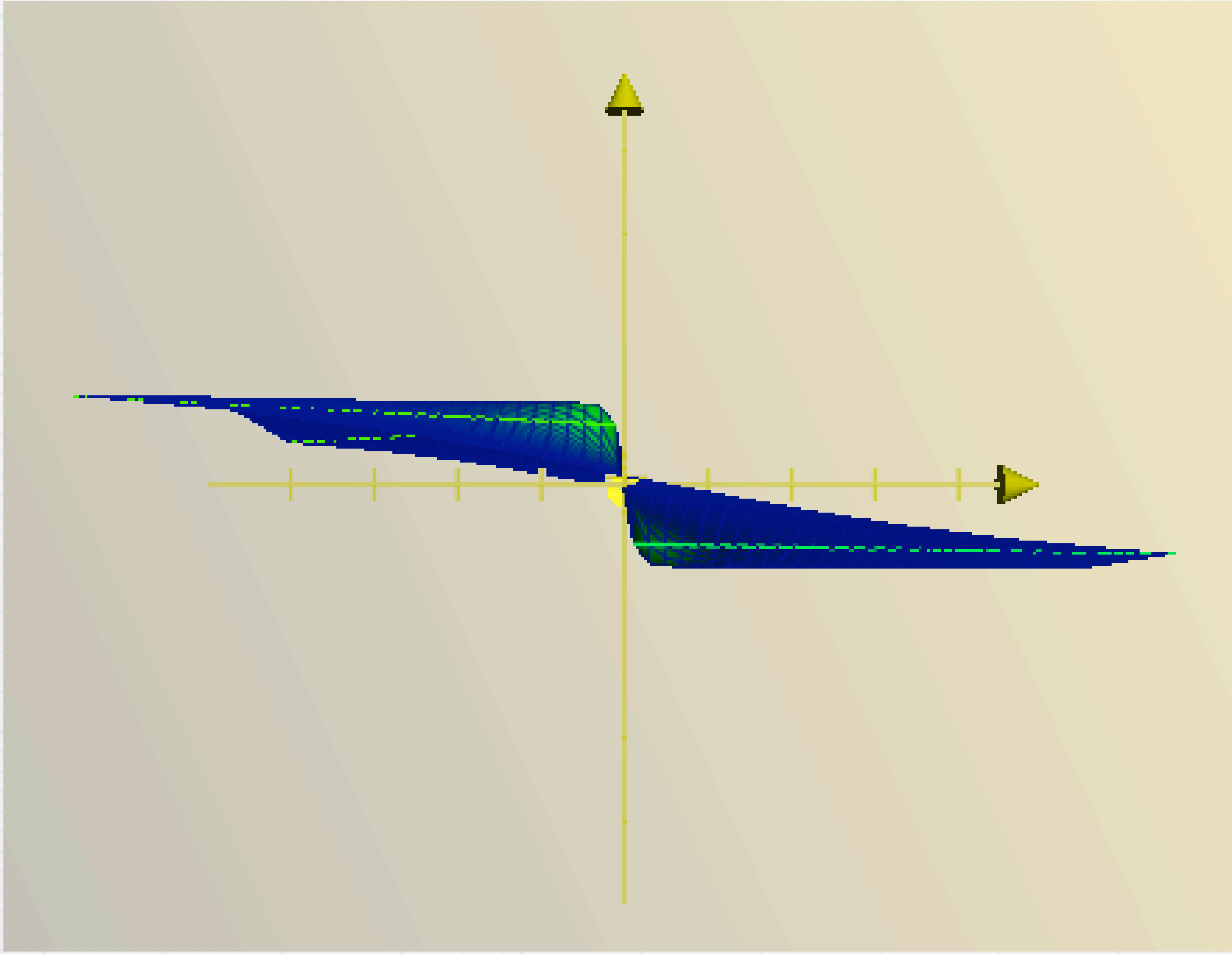
$$\lim_{(x,y)\to(0,0)} -\frac{x}{\sqrt{x^2+y^2}} = \lim_{x\to 0^-} -\frac{x}{\sqrt{x^2+(x)^2}}$$

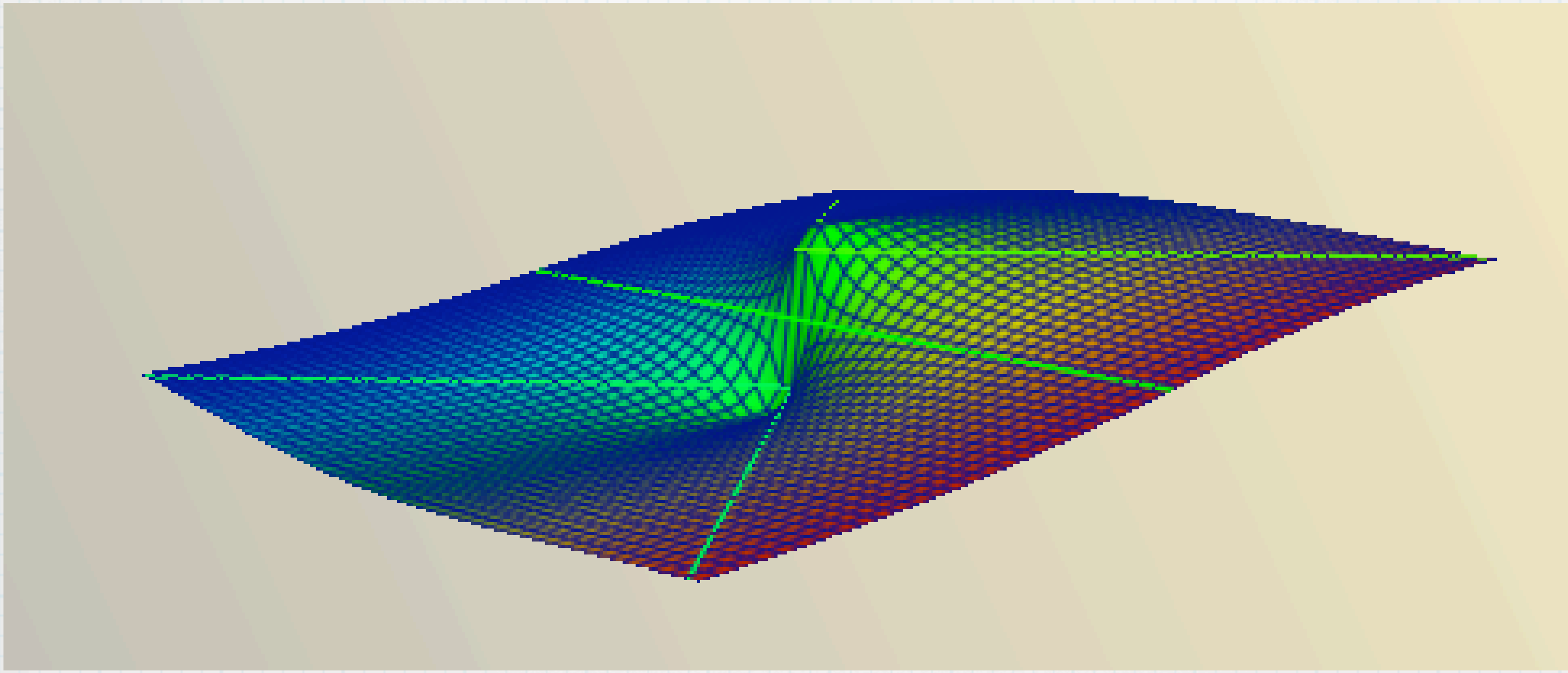
$$\lim_{x\to 0^-} -\frac{x}{\sqrt{x^2+(x)^2}} = \lim_{x\to 0^-} -\frac{x}{\sqrt{2}\,|x|} = \lim_{x\to 0^-} -\frac{x}{\sqrt{2}(-x)} = \lim_{x\to 0^-} \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

→ Limite no existe









c) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x\,y}{|x\,y|}$

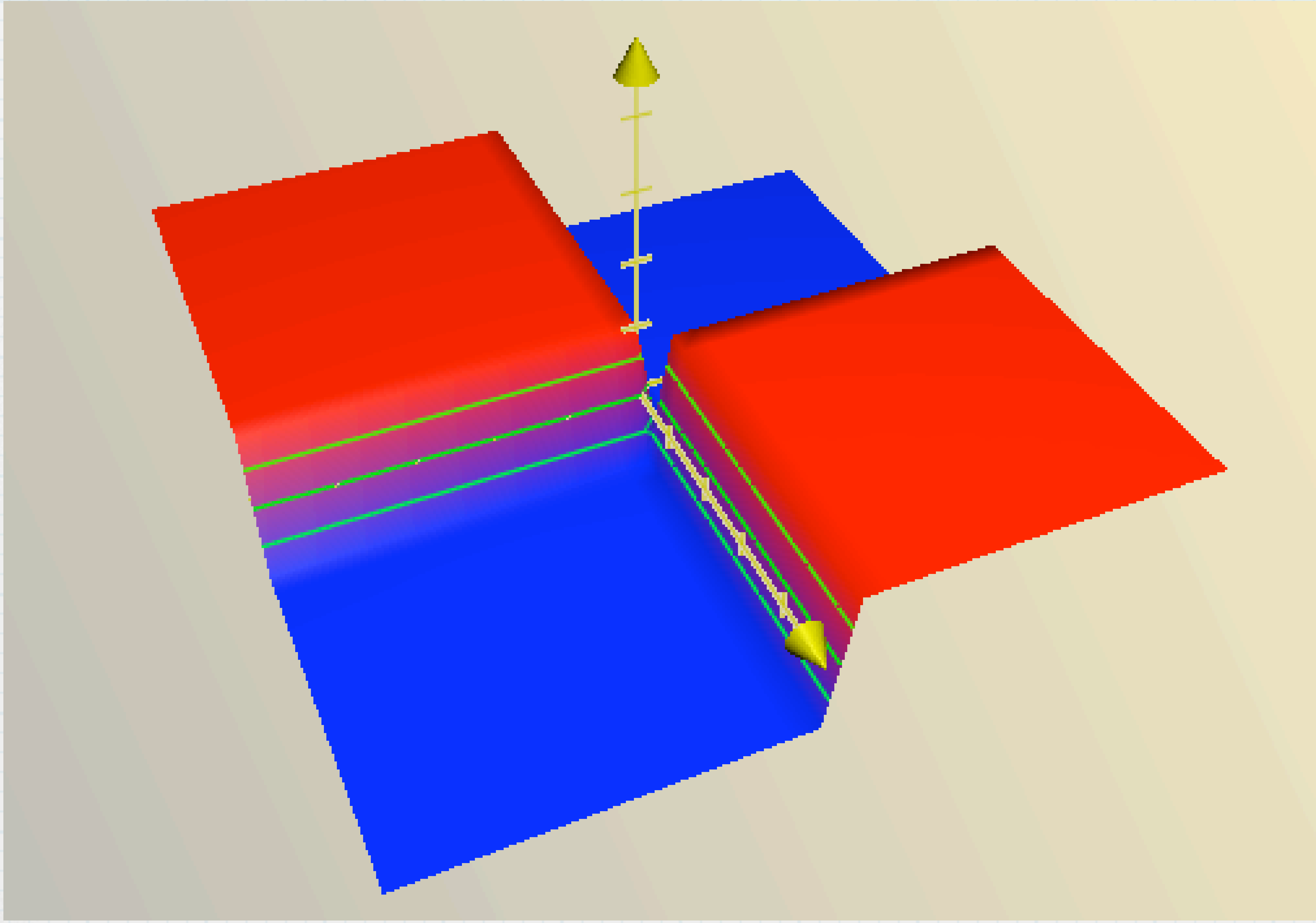
Trabajaremos a lo largo de la familia de curvas

$y = k\,x$ con $k \neq 0$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{x\to 0} \frac{x(kx)}{|x(kx)|}$$

$$\lim_{x\to 0} \frac{x(k\,x)}{|x(k\,x)|} = \lim_{x\to 0} \frac{k\,x^2}{|k\,x^2|} = \lim_{x\to 0} \frac{k}{|k|}$$

Este limite no tiene un valor único, en consecuencia no existe.



b)